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17MAT11

First Semester B.E. Degree Examination, Aug./Sept. 2020
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Obtain the n^{th} derivative of $\dfrac{x}{(x-1)(2x+3)}$. **(06 Marks)**

b. Find the angle intersection of the curves $r = a(1+\sin\theta)$ and $r = a(1-\sin\theta)$. **(07 Marks)**

c. Find the radius of curvature for the curve $x^3 + y^3 = 3axy$ at $\left(\dfrac{3a}{2}, \dfrac{3a}{2}\right)$. **(07 Marks)**

OR

2 a. If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(06 Marks)**

b. Obtain the pedal equation of the curve,
$\dfrac{2a}{r} = (1+\cos\theta)$. **(07 Marks)**

c. Find the radius of curvature for the curve $r^n = a^n \cos n\theta$. **(07 Marks)**

Module-2

3 a. Obtain Taylor's series expansion of log(cosx) about the point $x = \dfrac{\pi}{3}$ upto the fourth degree term. **(06 Marks)**

b. If $u = \sin^{-1}\left(\dfrac{x^2+y^2}{x+y}\right)$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \tan u$. **(07 Marks)**

c. If $u = \dfrac{yz}{x}$, $v = \dfrac{zx}{y}$, $w = \dfrac{xy}{z}$ show that $\dfrac{\partial(u,v,w)}{\partial(x,y,z)} = 4$. **(07 Marks)**

OR

4 a. Evaluate $\lim\limits_{x\to 0}\left(\dfrac{\tan x}{x}\right)^{\frac{1}{x}}$. **(06 Marks)**

b. Obtain the Maclaurin's expansion of the function $\log(1+x)$ upto the term containing x^4. **(07 Marks)**

c. If $u = f\left(\dfrac{x}{y}, \dfrac{y}{z}, \dfrac{z}{x}\right)$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0$. **(07 Marks)**

Module-3

5 a. A particle moves on the curve,
$x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$
where t is the time. Find the components of velocity and acceleration at time t = 1 in the direction of $i - 3j + 2k$. **(06 Marks)**

b. Show that $\vec{F} = (y+z)i + (z+x)j + (x+y)k$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

c. Prove that div(curl $\vec{A}$) = 0 **(07 Marks)**

OR

6 a. If $\vec{F} = (x+y+1)\hat{i} + \hat{j} - (x+y)\hat{k}$ show that $\vec{F}.\text{curl}\vec{F} = 0$. **(06 Marks)**

b. Find div $\vec{F}$ and curl $\vec{F}$, where $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$. **(07 Marks)**

c. Prove that $\text{curl}(\text{grad}\phi) = 0$. **(07 Marks)**

Module-4

7 a. Evaluate $\int_0^a x\sqrt{ax - x^2}\,dx$. **(06 Marks)**

b. Solve $(4xy + 3y^2 - x)dx + x(x + 2y)dy = 0$ **(07 Marks)**

c. Find the orthogonal trajectories of the family of curves $r = a(1 + \sin\theta)$. **(07 Marks)**

OR

8 a. Find the reduction formula for $\int_0^{\frac{\pi}{2}} \cos^n x dx$. **(06 Marks)**

b. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2 x$. **(07 Marks)**

c. A body in air at 25°C cools from 100°C in 1 minute. Find the temperature of the body at the end of 3 minutes. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix,
$$\begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$
(06 Marks)

b. Find the largest Eigen value and the corresponding Eigen vector of the matrix.
$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$
by power method taking the initial eigen vector as $[1 \quad 0 \quad 0]^T$ perform five iterations. **(07 Marks)**

c. Show that the transformation,
$y_1 = 2x + y + z$, $y_2 = x + y + 2z$, $y_3 = x - 2z$ is regular. Find the inverse transformation. **(07 Marks)**

OR

10 a. Solve the following system of equations by Gauss-Siedel method.

$20x + y - 2z = 17$

$3x + 20y - z = -18$

$2x - 3y + 20z = 25$

Carryout three iterations. **(06 Marks)**

b. Reduce the matrix,

$A = \begin{bmatrix} -1 & 3 \\ -1 & 4 \end{bmatrix}$ to the diagonal form. **(07 Marks)**

c. Reduce the following Quadratic form, $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form by orthogonal transformation. **(07 Marks)**

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